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AIR FORCES ON AIRFOILS MOVING FASTER THAN SOUND.

By J. Ackeret.

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AIR FORCES ON AIRFOILS MOVING FASTER THAN SOUND.*

By J. Ackeret.

Recent aircraft propellers not infrequently attain peripheral velocities approaching the velocity of sound. Mr. S. Albert Reed,** an American, has, in fact, exceeded this critical limit in experiments.

Professor Prandtl, in a still unpublished lecture, before the Göttingen "Seminar für angewandte Mechanik," gave approximation formulas for the region below the velocity of sound, which enable, in a very simple manner, the estimation of the effect of the compressibility. The approximation ceases, however, when the velocity w reaches the velocity of sound c ($w/c = 1$).

It is an interesting fact that a clear image can be obtained for still greater speeds by making a few obvious simplifications. We are undertaking the task of computing the air forces on a slightly cambered airfoil in the absence of friction and with an infinite aspect ratio. We also assume in advance that the leading edge is very sharp and that its tangent lies in the direction of motion. It does not appear to be impossible to get along without this last restriction. I do not know, however, of any utilizable device for a blunt leading edge.

The fact that the field of flow separates into smooth and

turbulent regions is characteristic for all speeds above the ve-
locity of sound. Since a weak disturbance can be propagated
only at the velocity of sound, it does not affect a current mov-
ing faster than sound and makes itself in no way noticeable in
a given field. I only need to recall the well known beautiful
pictures of flying projectiles, which Cranz took according to
the method of Mr. Mach (Fig. 1). The flow is disturbed inside
a cone with an angle of opening of 2α, but not at all disturbed
outside of this angle. The Mach angle α is calculated in known
manner from

$$\sin\alpha = c/w$$

and is therefore inversely proportional to **w**.

Important for our task, is a two-dimensional solution of
the flow equations, which was originated by Prandtl and Meyer
and which has already received much attention in technical cir-
cles, namely, the flow about an edge (Fig. 2).* I will discuss
it here only in so far as it concerns what is to follow. Imagine
the air flowing out of a very large opening. If the pressure
p_0 before the constriction is at least double the outside pres-
sure p_1, the velocity of sound in the opening is then

$$c = \sqrt{g\, k\, R\, T_1}$$

$k = c_p/c_v$ = ratio of specific heats, R = gas constant, T = ab-
solute temperature, g = accleration due to gravity.

* Prandtl, "Gasbewegung" in "Handbuch der Naturwissenschaften,"
Vol. 4, Jena, 1913. Th. Meyer, "Mitteilung über Forschungsarbei-
ten, 1908, No. 62. Stodola Dampf- und Gasturbinen, Edition 6,
p. 810.

The air flows with　c　over the board. Below the latter, another pressure　p_2　may reign, which is assumed to be smaller than　p_1. It seeks to make itself noticeable as a backward disturbance around the edge, but does not affect the flow. The air flows unhindered as far as the edge, at which point the pressure must pass from　p_1 to p_2. An adiabatic flow, free from eddies, is assumed at the beginning of the bending of the streamlines. The vortex laws of Helmholtz and Kelvin hold good also for compressible flows, if they take place adiabatically and there are no condensation impulses (shock waves). It is noteworthy that all condition and velocity magnitudes have constant values on one and the same radius. If the counterpressure　p_2　is reached, the bending ceases and the flow continues, as a free stream limited on one side, around the angle　ν. The pressure　p_2　is reached everywhere on the intersection points of a line passing through the edge (with the angle　φ_2). We accordingly have three plainly divided regions of flow.

Regarding the derivation of the equation for the lines of flow, etc., we refer to Mr. Meyer's article. The numerical values in Fig. 3 were partly taken from Mr. Meyer's article. For us the pressure ratio　$p : p_0$, as plotted against the angle of deflection　ν, is important. We there find also the curve for p_0 plotted for the case when　p　equals　1 atm. = 10000 kg/m², which pressure we will consider, in what follows, as the pressure in the undisturbed region.

The application to the airfoil (Figs. 4-5) is made as fol-
lows: The air has a velocity above that of sound (over 340 m/s).
The leading edge will cause a slight disturbance, which is prop-
agated into outside space at the angle of Mr. Mach. If the sur-
face is tangential to the direction of flow and the edge very
sharp, the disturbance is then infinitely small and the thermal
process reversible (Stodola, "Dampf- und Gasturbinen," Edition 6,
p. 835). We will now consider the upper surface of the airfoil
as a "developed" and enlarged Prandtl edge. The deflection,
which there takes place in a very small space, is here distrib-
uted over the depth t of the airfoil. There is assumed to be
constant pressure and a constant deflection v on every Mach
line. The magnitude of v is determined, however, by the incli-
nation of the surface to the original flow direction at the point
where the corresponding Mach line originates. If we now take
from Fig. 3 the pressure p corresponding to the values of v ,
then, as already mentioned, if we disregard the effects of fric-
tion, the determination of the resultant power on the airfoil is
reduced to a simple integration problem. On the upper side of
the airfoil the air is expanded, while it is compressed on the
lower side. It is therefore important that the pressure increase
should not take place too rapidly on the lower side, so that the
Mach waves will not meet near the airfoil. Such an encounter
corresponds to a compression shock, which may react, if too near
the surface of the airfoil, by producing new disturbance waves.

Obviously the flow velocity and the airfoil depth (chord) will determine whether these waves will again strike the airfoil surface and change the pressure distribution. We must also determine the intersection points of infinitely close Mach waves.

In Figs. 4 and 5, the pressure distributions of Fig. 3 are taken and a few waves plotted, somewhat as they could actually be seen, if the surface should show rough places. In fact, the process must take place continuously, as in the Meyer flow. The lines of flow can be plotted without difficulty if we read from Fig. 3 the angle ψ between the perpendicular to the direction of the Mach waves and the direction of flow. From the field of direction thus found, we obtain the picture of the lines of flow. The angle of the Mach waves with the surface is $90° - \psi$. Regarding all these angles, they must be calculated from the outward rate of flow. If, for example, the air flows at 600 m (1968 ft.) per second, it has, as it were, according to Fig. 3, already flowed around an edge and experienced a deflection of about $20°$. Consequently, υ is increased a few degrees on the negative-pressure side of the airfoil but is diminished just as much on the positive-pressure side.

We now introduce a simplification by assuming all deflections to be small. Then we can replace the curve p/p_0, piece by piece, by the tangents of these deflections. The pressure differences are then proportional to the changes in the angle of deflection $\Delta\upsilon$. The pressure at a given point is

$$p = p_1 \mp p_0 \; \frac{d \left(\frac{p}{p_0} \right)}{d \, v} \; \Delta v \; ; \; \Delta v = \frac{d \, y}{d \, x} \quad ^*$$

in which p_1 denotes the pressure at the entrance edge. The upper sign $(-)$ applies to the negative-pressure side and the lower sign $(+)$ to the positive-pressure side. If we write, for short

$$\frac{d \left(\frac{p}{p_0} \right)}{d \, v} = \zeta$$

then the lift $= A = \int p \, d \, x$ over the whole surface (for the span 1)

$$= 2 \, p_0 \, \zeta \int \Delta v \, d \, x = 2 \, p_0 \, \zeta \int \frac{d \, y}{d \, x} \, d \, x = 2 \, p_0 \, \zeta \, h$$

We therefore arrive at the simple conclusion that the lift depends only on the height h (Fig. 4) and not on the special shape of the surface. The drag, on the contrary, is

$$W = \int p \, d \, y = 2 \, p_0 \, \zeta \int \frac{d \, y}{d \, x} \, d \, y = 2 \, p_0 \, \zeta \int \left(\frac{d \, y}{d \, x} \right)^2 d \, x.$$

In Figs. 4 and 5, an airfoil of parabolic shape is assumed, $y = -a \, x^2$. For this special case, we obtain the drag

$$W = 2 \, p_0 \, \zeta \int 4 \, a^2 \, x^2 \, d \, x = \frac{8}{3} \, p_0 \, \zeta \, a^2 \, t^3 = \frac{8}{3} \, p_0 \, \zeta \, \frac{h^2}{t}$$

and the lift-drag ratio

$$\frac{A}{W} = \frac{3}{4} \, \frac{t}{h} \, .$$

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* It is probably clear, that we must take v in absolute mass ($57.3^0 = 1$).

From the simplifications, there follows a linear increase or decrease of pressure toward the rear (dash lines). If we take the pressure at each point from Fig. 3, we obtain the plain pressure lines plotted in the lower part of Figs. 4-5. The difference is insignificant. With a different profile, we would naturally obtain a different pressure distribution. The resultant force is applied at a point about 2/3 of the chord from the leading edge.

We have thus found that, even with frictionless flow and no induced drag, there is a residual drag (wave resistance). It is therefore probable that the employment of velocities above that of sound has no aerodynamic advantage.

We can, as customary, apply the calculated values to the so-called dynamic pressure $q = (\rho/2)\,w^2$, although this conception here loses its value, because the Bernouilli equation no longer holds good in its simple form, due to the finite compressibility:

and for Fig. 5,
$$c_a = 0.12; \quad c_w = 0.008$$
$$c_a = 0.077; \quad c_w = 0.005.$$

We see, therefore, that the coefficients change as the velocity increases. The reason for this lies partly in the definition of dynamic pressure, which, as can be seen from Fig. 3, continually recedes from the definitive pressure p_0. Secondly,

$\zeta = \dfrac{d\left(\dfrac{p}{p_0}\right)}{dv}$ also changes greatly with the velocity. In Fig. 6a,

the course of the coefficients is plotted against the ratio
w : c. In Fig. 6b there is shown, on the right, a curve of the
resistance figures for a rifle bullet, according to the measure-
ments of Cranz and Becker. We see a qualitative agreement in
the decrease of the coefficients at high velocities. No defi-
nite conclusions can be drawn concerning the applicability of the
assumptions, so long as there is no experimental confirmation.
It may also be noted that a numerical calculation is possible
without any arbitrary constants.

' We have not yet made any computations concerning the phenom-
ena behind the airfoil. On considering the pressure curves, we
see that the positive pressure and negative pressure come close
together on the trailing edge. A Meyer back-expansion takes
place on the positive side, but a compression shock, emanating
from the same, is to be expected on the negative side. These
phenomena can have no effect on the pressure distribution, since
the possible disturbances do not make themselves noticeable to-
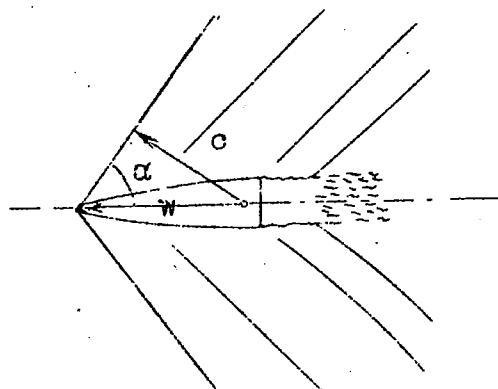ward the front.

Fig.1   Mach waves on a flying projectile.

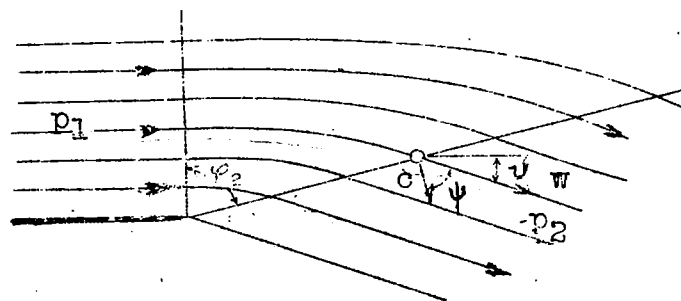
Fig.2   Flow about an edge according to
        Prandtl & Meyer.
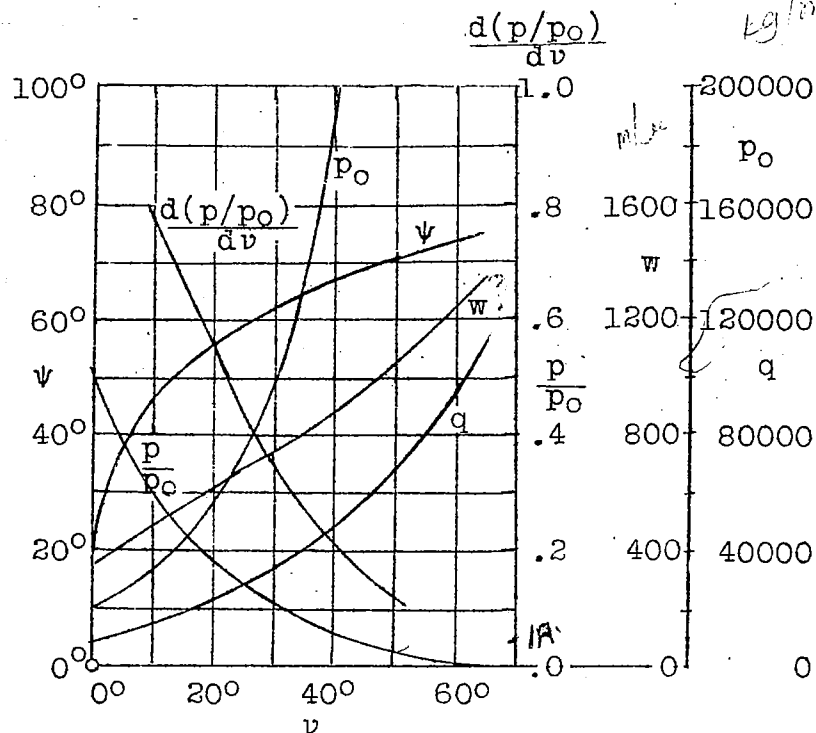   The lines of flow are not limited above.

Fig.3   Pressure, velocity, etc. plotted against
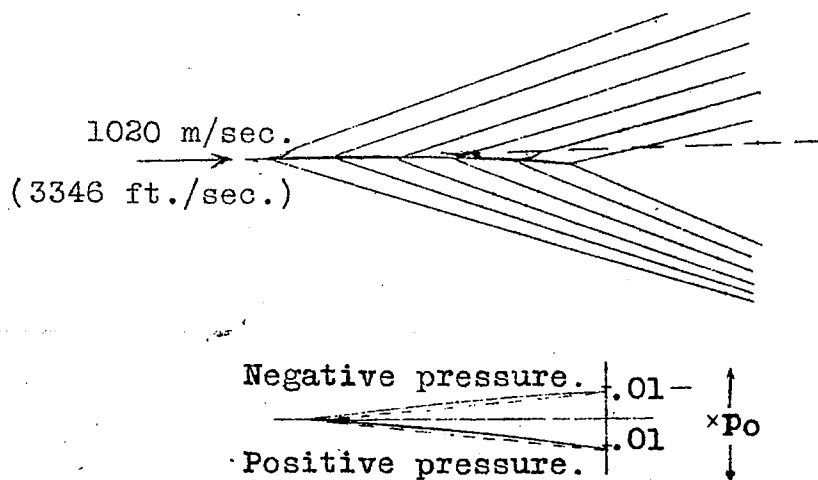   the deflection angle.

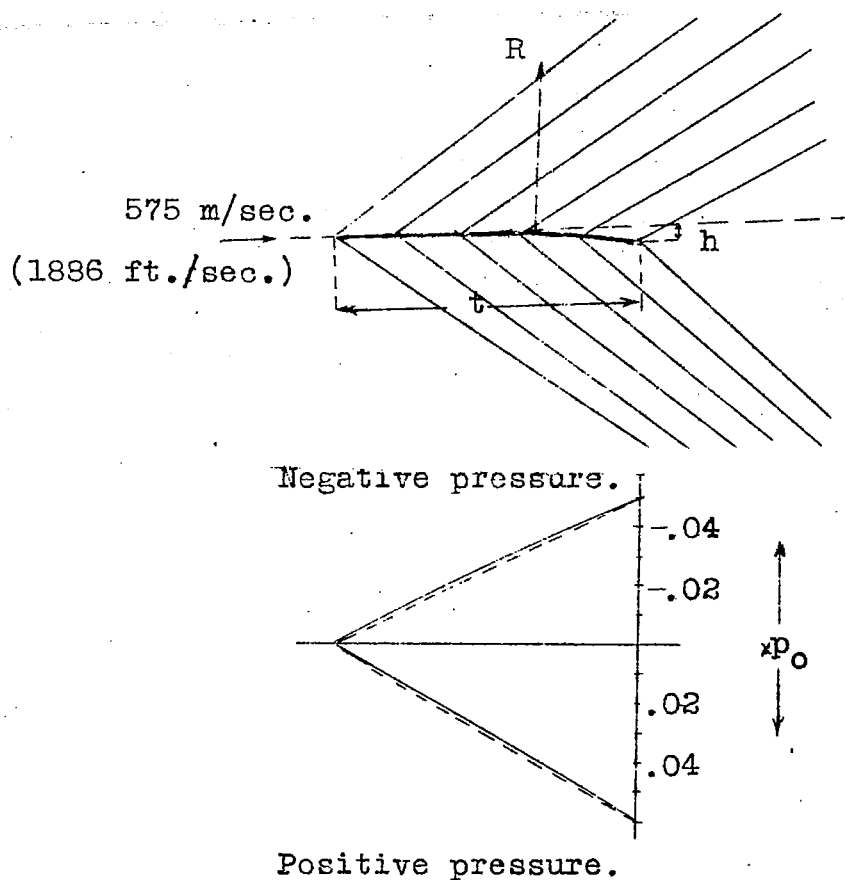Fig.5   Mach waves at 3 times the
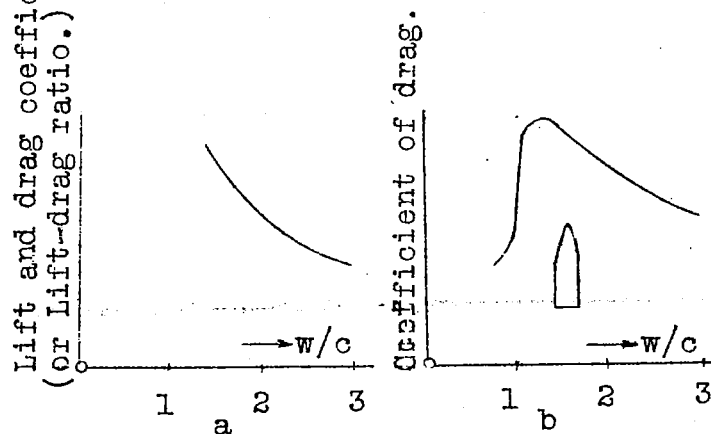   velocity of sound.

Fig.4   Mach waves on a cambered surface.

Fig.6   Coefficients plotted against velocity.